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Pension System Sustainability and Public Debt*

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Abstract

This paper presents consideration of a pension system not as managed using a balanced budget in each period but rather with a budget system in which issuance of public bonds is allowed. In an endogenous fertility model, these analyses reveal how a child allowance affects the stock of public debt per capita and whether the government can maintain constant benefit and constant contribution rates or not while maintaining fiscal sustainability. Results obtained for a small open economy reveal that child allowances can always bring about a steady state with constant public debt per capita. However, child allowances can not always bring about a steady state in a closed economy. Moreover, this paper presents examination of whether child allowances can reduce public debt per capita or not. In both a small open economy and a closed economy, child allowances are not always sufficient to reduce the public debt per capita. According to numerical examples given by the realistic parameter condition, a steady state equilibrium with public debt or public asset can be obtained; child allowances can reduce the public debt per capita (or raise the public asset). However, that steady state equilibrium is not locally stable. Other parameter conditions have a stable steady state in which child allowances can raise public assets per capita.

Keywords: Pension, Public debt, Endogenous fertility, Child allowances

JEL Classifications:J13, H55, G23

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1 Introduction

In Japan, the share of the population older than 65 years old among the total population, which reached 20.1% in 2005 [1], has continued increasing. Nevertheless, total fertility in Japan has remained low. [2] Because of this rapidly aging society with fewer children, which decreases pension contributions concomitantly with the decrease in younger people, and because of the rapid increase in older people (with increasing pension benefits), the pension system in Japan was reformed in 2004. Those reforms increased the contribution rate and national tax burden, with the intention of eventually fixing the contribution rate at some future date. [3] The government also applies a macroeconomic sliding scale by which it examines economic and social factors (e.g., aging, fewer children) and controls benefits. We regard the public pension system in Japan, reformed as described above, as a Defined Contribution (DC) system.

Recently, social security payments became the greatest ever recorded in recent national fiscal budgets in Japan.[4] Payments for social security include the national tax burden for pension benefits, which continue to increase. [5] The annual expenditure can not be financed using tax revenues alone. Therefore, the government issues public debt. The stock of public debt continues to increase; it has reached 600 trillion Japanese yen.[6] The government can not collect tax revenue and contribute it to provision of pension benefits. Therefore, the government must issue public debt. However, even if the government were allowed to continue its issuance of public bonds, the government would be compelled eventually to increase tax revenues to avoid bankruptcy through an increased contribution rate, a decrease in pension benefits, or child support policies to halt the decrease in birth rates.

This paper describes a model incorporating children as consumption goods and examines a pension system allowing the issuance of public bonds. Many analyses described in earlier papers have subsumed a balanced budget pension. Because of a balanced budget, the government must change the benefit

[1]Data: 'A National Census in Japan'

[2]Although the total fertility rate in Japan decreased to 1.26 in 2005, the fertility rate increased slightly to 1.37 in 2008. The total fertility rate in Japan remains less than that in either France or Sweden (Data: Ministry of Health, Labour and Welfare 'Vital Statistics').

[3]Before reforming the pension system, the contribution rate of employees' pensions was 13.58%. Payments to the national pension were 13.3 thousand Japanese yen. Following the 2004 pension reform, payments will increase step-by-step. Finally in 2017, the contribution rate will be 18.3% and the payment of the national pension will be 16.9 thousand Japanese yen.

[4]Social security payments' share in the original national fiscal budget in 2009 was 28%.

[5]Social security expenditures increase every year (Data: Fiscal in Japan). In the National General Account for fiscal year 2008 in Japan, social insurance accounts for 80.4% of social security expenditures.

[6]According to the Ministry of Finance, Japan, the stock of public debt reached 581 trillion Japanese yen in fiscal year 2009.

or contribution rate to balance the budget. However, such changes might be not desired because of unexpected benefits or contributions or intergenerational conflict. Pension benefits or contribution rates need not be changed if the government can continue financing its obligations using public debt.

Some earlier papers have presented examinations of pension systems. Pension analyses include two issues: One for the difference between Defined Contribution (DC) and Defined Benefit (DB) and the other for a relation between the pension system and endogenous fertility. Borgmann (2005) compared DC with DB and ascertained which pension system is better. Different from a Defined Benefit (DB) system, the pension benefit in DC depends on the dependency ratio and economic growth, which includes uncertainty because older people can not know the benefit in advance. Yasuoka and Oshio (2008) examined DC and derived an optimal contribution rate given an uncertain benefit. These models incorporate the assumption of exogenous population growth.

Some papers have presented examination of the pension system in an endogenous fertility model. Zhang and Zhang (2007) examined the optimal contribution rate for a pension in an endogenous fertility and DC pension model. Oshio and Yasuoka (2009) set an endogenous fertility model that included a DB pension. With an increase in the number of older people with fewer children, the contribution rate increases. Therefore, fertility decreases because the contribution rate increases. Finally, fertility continues decreasing over time. For that reason, such a pension system is not sustainable. Lin and Tian (2003) also examined different means to finance the pension benefit in DB.[7] These papers all present examinations of balanced budget systems. Considering public debt, the government can fix not only pension benefits but also pension contributions. The pension system with fixed pension benefits and contributions in a public debt model are not examined. Therefore, our paper presents an examination of them.

Some earlier works have presented examinations of a pension system and a child allowance with an endogenous fertility model that incorporates children as consumption goods. Oshio (2001), van Groezen, Leers, and Meijdam (2003), and van Groezen and Meijdam (2008) showed that the fertility level that is chosen by households is lower than the socially optimal fertility in a pay-as-you-go pension system.

[7] However, it is noteworthy that Lin and Tian (2003) and Oshio and Yasuoka (2009) regarded children as investment goods. We should examine the difference of motivation to have children in examining pension and fertility. As explained by Nishimura and Zhang (1992) and Zhang and Zhang (1998), by virtue of pensions, parents limit their number of children because a pension guarantees an income during their old period on behalf of their children. However, if children are regarded as consumption goods, then an increase in income by pensions raises fertility (e.g., Oshio (2001), van Groezen, Leers, and Meijdam (2003)).

Moreover, they asserted, based on those results, it is important to provide child allowances to raise fertility. [8] With a balanced pension budget, an increase in fertility increases future pension benefits. However, households do not consider this positive externality effect. Therefore, a child allowance is necessary. In fact, a child allowance has been adopted to stop the trend of fewer children in developed countries. Halting that trend toward fewer children pulls up future tax revenues through increased working population size. Examining whether a child allowance can raise fertility and how a child allowance affects the dynamics of public debt is important. Our paper presents consideration of a child allowance financed by public debt without an increase in the tax burden and examines whether a child allowance can raise fertility or not and whether public debt per capita is reduced or not because of an increase in population size.

Diamond (1965) considered public debt in an overlapping-generations model. Samuelson (1958) and Azariadis (1993) examined whether a fiscal policy that brings about fiscal deficit is sustainable or not. Sustainability depends on the primary fiscal deficit and the gap separating interest rate and population growth rate. Government expenditure in these models is regarded as public consumption. Similarly, Chalk (2000) and Bräuninger (2005) considered whether public bond financing is sustainable or not in the model incorporating government consumption.[9] Ono (2003) examined a pension system with public debt. Ono (2003) fixed the contribution rate and benefit and issued public debt to finance the wedge between contributions and benefits in a closed economy. The dynamics of public debt and capital can be shown to depend on parametric conditions and the initial public debt. Meijdam, van de Ven, and Verbon (1996) examined the dynamics of public debt in a small open economy and derived the mechanism by which taxation affects the dynamics of public debt. This paper presents consideration of public debt introduced by Meijdam, van de Ven, and Verbon (1996) and Ono (2003). However, in addition to public debt, we set an endogenous fertility model.

Our paper presents examination not only of a small open economy but also of a closed economy. In a small open economy, even if the public debt continues to increase because of a primary deficit, the stock of public debt converges to a constant level by virtue of the child allowance. The child allowance raises fertility. Therefore, the size of the younger population is large: tax revenues increase and the

[8] However, Fanti and Gori (2009), which does not incorporate pensions, showed that taxation for children raises fertility because of the income effect in a closed economy. Simultaneously, this result signifies that a child allowance lowers fertility because of decreased capital per capita, i.e. income.

[9] Yakita (2008) investigated public capital formation financed by public debt and examined the sustainability of public debt.

increased population decreases the public debt per capita directly. Meijdam, van de Ven, and Verbon (1996) analyzed the dynamics of public debt in a small open economy. Meijdam, van de Ven, and Verbon (1996) showed that public debt per capita continues increasing because of a decrease in the tax burden. However, analyses described herein show that an increase in government expenditure as a child allowance can stop increasing and public debt per capita converges to a certain level.

The public debt decreases physical capital supplied to the product sector, which raises the interest rate in a closed economy. This is a crowding-out effect. An increase in public debt for a child allowance raises the interest rate and enlarges the primary deficit. Therefore, even if a child allowance raises tax revenue and decreases public debt per capita directly, a child allowance can not always stop increasing public debt because of an increase in the interest rate.

The numerical examples given by the realistic parameter conditions show a stable steady state equilibrium in which child allowances can reduce the public debt per capita (or raise the public asset). However, this steady state equilibrium is not stable. If other parameter conditions are given, then we obtain the stable steady state equilibrium with positive public assets, showing that child allowances can raise public assets. Our paper describes the possibility that child allowances financed by a public asset, that is pension funds, can increase public assets. Ono (2003) examined how aging affects the dynamics of public debt per capita in exogenous fertility and a closed economy and showed that the effects of aging on public debt are ambiguous.[10] Our paper is intended to examine whether fertility that is increased by a child allowance as a child-care policy halts the increase of public debt or not.

The remainder of this paper is the following. Section 2 of this paper establishes the model. Section 3 derives equilibrium in a small open economy and closed economy. Section 4 examines policy effects such as an increase in the child allowance in a closed economy with numerical examples. The final section presents results.

2 The Model

This model economy consists of a two-period (young and old) overlapping generations model with three agents: households, firms, and a government. In the following subsection, we explain each agent.

[10] The effect of an increase in the population growth rate is ambiguous, too.

2.1 Households

Each household lives in three periods—childhood, young, and old—and supplies labor to earn an income during the young period. Young people supply labor inelastically for consumption during the young period and use savings for consumption during the old period in addition to caring for children. A government provides not only a pension system that gives older people a fixed benefit but also a child allowance for younger people. The budget constraint is given as

$$c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} + (z_t - q_t)n_t = (1-\tau)w_t + \frac{p_{t+1}}{1+r_{t+1}}. \tag{1}$$

Therein, q_t denotes the child allowance. Furthermore, n_t represents the number of children. Necessary goods to bring up a child are represented as z_t. In addition, c_{1t} and c_{2t+1} respectively denote consumption during young and old periods. Here, w_t shows the wage rate; interest rate $1 + r_{t+1}$ is given for savings. Younger people face income taxation (tax rate or contribution rate τ). Older people receive pension benefit p_{t+1}. Furthermore, t signifies the period. We assume that the child-care cost z_t depends on wage income such as $z_t = \hat{z}w_t$ ($\hat{z} > 0$).[11] Moreover, the government provides a child allowance as $q_t = \hat{q}w_t$ ($\hat{z} > \hat{q} > 0$) and pension benefit as $p_{t+1} = \hat{x}w_t$ ($\hat{x} > 0$).[12] A household's utility function is assumed as

$$u_t = \alpha \ln n_t + \beta \ln c_{1t} + (1-\alpha-\beta)\ln c_{2t+1}, \ 0 < \alpha, \beta < 1, \ \alpha + \beta < 1. \tag{2}$$

Under the budget constraint (1), the allocation of c_{1t}, c_{2t+1} and n_t to maximize their utility is shown as

$$c_{1t} = \beta w_t \left((1-\tau) + \frac{\hat{x}}{1+r_{t+1}} \right), \tag{3}$$

$$c_{2t+1} = (1+r_{t+1})(1-\alpha-\beta)w_t \left((1-\tau) + \frac{\hat{x}}{1+r_{t+1}} \right), \tag{4}$$

$$n_t = \frac{\alpha \left((1-\tau) + \frac{\hat{x}}{1+r_{t+1}} \right)}{\hat{z} - \hat{q}}. \tag{5}$$

[11] van Groezen, Leers, and Meijdam (2003), Fanti and Gori (2009) and Oshio (2001) also assumed the same child-care cost. van Groezen and Meijdam (2008) examined an economy with child-care cost z as a wage increasing function.

[12] Zhang and Zhang (2007) explained that the assumed pension benefit is practiced by many developed countries such as France and Germany. $\hat{x}$ denotes replacement rate.

2.2 Firms

A representative firm produces final good Y_t with constant returns to scale or a neoclassical product function, shown as

$$Y_t = K_t^{\theta} N_t^{1-\theta},\ 0 < \theta < 1. \tag{6}$$

The firm inputs capital stock K_t and labor (population size of younger people) N_t. With a perfectly competitive market, the wage rate w_t and the interest rate r_t are

$$w_t = (1-\theta)k_t^{\theta}, \tag{7}$$

$$1 + r_t = \theta k_t^{\theta-1}, \tag{8}$$

where $k_t \equiv \frac{K_t}{N_t}$ and the capital stock depreciates fully in a single period.

2.3 Government

The government executes two policies: one for the pension and the other for a child allowances. A payroll tax rate τ, which we can regard as the contribution rate, is levied on younger people. Older people receive pension benefit p_t. Assuming a balanced budget in each period, the government must change the tax rate to balance the budget. However, allowing issuance of public debt, the government need not change the tax rate in each period. The governmental budget is shown as[13]

$$b_{t+1} = \frac{1+r_t}{n_t} b_t + \frac{\hat{x} w_{t-1}}{n_t n_{t-1}} + \hat{q} w_t - \frac{\tau w_t}{n_t}. \tag{9}$$

3 Equilibrium

Before deriving the equilibrium in closed economy, we derive the equilibrium in a small open economy.

3.1 Small Open Economy

In a small open economy, the wage rate w_t and interest rate $1 + r_t$ are fixed as $\hat{w}$ and $1 + \hat{r}$. Then, the dynamics of the public debt stock per capita (9) can be shown as

$$b_{t+1} = \frac{1+\hat{r}}{\hat{n}} b_t + \frac{\hat{x}\hat{w}}{\hat{n}^2} + \left(\hat{q} - \frac{\tau}{\hat{n}}\right)\hat{w}, \tag{10}$$

[13] See Appendix for a detailed proof.

where $\hat{n} = \frac{\alpha\left(1-\tau+\frac{\hat{x}}{1+\hat{r}}\right)}{\hat{z}-\hat{q}}$. Even if $1+r$ is larger than n and the primary balance is in deficit, $\frac{\hat{x}\hat{w}}{\hat{n}^2} + \left(\hat{q} - \frac{\tau}{\hat{n}}\right) > 0$, the public debt stock b_t converges to $\hat{b}$ because of child allowances as a result of $\frac{\partial \hat{n}}{\partial \hat{q}}$, as shown in Fig. 1.

[Insert Fig. 1 around here.]

If the public debt per capita converges to $\hat{b}$, then the child allowance $\hat{q}$ can decrease $\hat{b}$ if the following condition holds:

$$\frac{d\hat{b}}{d\hat{q}} = \frac{\hat{n}\left(\hat{w} + \frac{\left(\hat{q}-\frac{\hat{x}}{\hat{n}^2}\right)\hat{w}-\hat{b}}{\hat{z}-\hat{q}}\right)}{\hat{n}-1+\hat{r}} < 0. \tag{11}$$

Meijdam, van de Ven, and Verbon (1996) showed that the public debt stock per b_t continues increasing because of a decrease in tax revenue. An increase in the child allowance increases the government expenditure under constant tax revenue, thereby it is considered that b_t does not converge to a certain level. However, child allowances raise fertility $\hat{n}$; then b_t converges to a certain level even if government expenditures are expanded.[14]

However, this result is derived in a small open economy, which is not considered private capital accumulation. If the public debt stock increases in a closed economy, then the private capital stock might decrease. This effect decreases the wage rate and increases the interest rate. Therefore, child allowances cannot halt the continual increase in the public debt per capita because tax revenue decreases and the interest rate increases. In the next subsection, we derive the equilibrium and examine whether child allowances can stop continue increasing public debt per capita and whether child allowances can decrease public debt per capita.

3.2 Closed Economy

The equilibrium in this model is specified by two dynamics equations: one for public debt per capita b_t and the other for capital stock per capita k_t. The dynamics of b_t can be given as (9). Considering n_t given by (5), we obtain the dynamics of b_t as a function of k_t. Defining s_t as household saving, the equilibrium of capital market is given as $b_{t+1} + k_{t+1} = \frac{s_t}{n_t}$. Then, the dynamics of k_t can be represented

[14] Theoretically, the discussion presented in this paper shows that fertility increases as far as the child allowances increase. However, we think that the fertility can not be raised more than the biological limit level. Therefore, if an interest rate is extremely high, then an increase in the fertility provided by child allowances does not contribute to convergence of public debt per capita.

as

$$b_{t+1} + k_{t+1} = \left[\frac{1-\tau}{n_t} - \frac{(\alpha+\beta)(\hat{z}-\hat{q})}{\alpha}\right] w_t. \tag{12}$$

Therefore, the equilibrium in this economy is specified by (5), (7), (8), (9), and (10). This equation shows the capital market equilibrium condition. Defining k and b as $k = k_{t+1} = k_t$ and $b = b_{t+1} = b_t$, k and b in the steady state are determined according to the following equations:

$$k + b = \left(\frac{1-\tau}{n} - \frac{(\alpha+\beta)(\hat{z}-\hat{q})}{\alpha}\right) w, \tag{13}$$

$$b = \frac{w\left(\frac{\hat{x}}{n} + \hat{q}n - \tau\right)}{n-(1+r)}. \tag{14}$$

Therein,

$$w = (1-\theta)k^{\theta}, \tag{15}$$

$$1 + r = \theta k^{\theta-1}, \tag{16}$$

$$n = \frac{\alpha\left[(1-\tau) + \frac{\hat{x}}{1+r}\right]}{\hat{z}-\hat{q}}. \tag{17}$$

Capital per capita k in the steady state is given such that the following condition holds:

$$\frac{k^{1-\theta}}{1-\theta} = \frac{1-\tau}{n} - \frac{(\alpha+\beta)(\hat{z}-\hat{q})}{\alpha} - \frac{\frac{\hat{x}}{n} + \hat{q}n - \tau}{n-(1+r)}. \tag{18}$$

This paper presents an examination of the proposition that child allowances financed by public debt can decrease the public debt stock in the long run because of the effect of increase in tax revenue brought about by younger people in the future period.

We examine how the respective dynamics of physical capital stock k_t and public debt b_t are determined. Calculating $\frac{dk}{d\hat{q}}$ and $\frac{db}{d\hat{q}}$ yields

$$\frac{dk}{d\hat{q}} = \frac{a_{22}b_1 - b_2}{a_{11}a_{22} - a_{21}}, \tag{19}$$

$$\frac{db}{d\hat{q}} = \frac{a_{11}b_2 - a_{21}b_1}{a_{11}a_{22} - a_{21}}, \tag{20}$$

where

$$\begin{aligned}
a_{11} &= 1 - \frac{k+b}{w}\frac{\partial w}{\partial k} + \frac{(1-\tau)w}{n^2}\frac{\partial n}{\partial k}, \\
a_{21} &= b\left(\frac{\partial n}{\partial k} - \frac{\partial r}{\partial k} - \frac{n-(1+r)}{w}\frac{\partial w}{\partial k}\right) + \left(\frac{\hat{x}w}{n^2} - \hat{q}\right)\frac{\partial n}{\partial k}, \\
a_{22} &= n - (1+r),
\end{aligned}$$

$$b_1 = \frac{\alpha+\beta}{\alpha} - \frac{1-\tau}{n^2}\frac{\partial n}{\partial \hat{q}},$$
$$b_2 = wn + \left(\hat{q} - \frac{\hat{x}w}{n^2} - b\right)\frac{\partial n}{\partial \hat{q}},$$

and where $\frac{\partial r}{\partial k} = \theta(\theta-1)k^{\theta-2}$, $\frac{\partial w}{\partial k} = \theta(1-\theta)k^{\theta-1}$, $\frac{\partial n}{\partial \hat{q}} = \frac{n}{\hat{z}-\hat{q}}$ and $\frac{\partial n}{\partial k} = \frac{\hat{x}\alpha\theta(1-\theta)k^{\theta-2}}{(\hat{z}-\hat{q})(1+r)^2}$. An increase in child allowance $\hat{q}$ directly raises public debt per capita b as shown by the first term wn in b_2. However, an increase in the child allowance affects public debt per capita b through the change of capital stock per capita k, as shown by a first term of the right-hand-side of (20). If the sign of $\frac{db}{d\hat{q}}$ is negative, then child allowances can reduce the public debt per capita because an increase in population size brings about additional tax revenue. However, we can not show the sign of $\frac{db}{d\hat{q}}$ and whether the steady state equilibrium is locally stable or not. Therefore, given some parametric condition, our paper presents an examination of whether the parameter set of $\frac{db}{d\hat{q}}$ exists or not in the following section.

4 Numerical Examples

This section presents an examination, with numerical examples, of whether child allowances can reduce public debt stock per capita or not. However, without parametric conditions, many cases show the positive sign of $\frac{db}{d\hat{q}}$. Therefore, we set some parameter conditions as follows. As reported by de la Croix and Doepke (2003), the discount rate of the utility from the consumption during the older period is set as about 0.3 because the quarterly discount rate is 0.99 and one period in the overlapping generations model is regarded as 30 years and calculated as 0.99^{120}. Therefore, we consider the weight of utility from consumption in old period is $1-\alpha-\beta = 0.3$, i.e., (i) $\alpha+\beta = 0.7$. Second, we set the capital income share (ii) $\theta = 0.3$ as observed in Japan. Third, the replacement rate of pension benefit in Japan is set as about fifty percent for the young generation's wage income. Therefore, we set (iii) $x = 0.5$. Fourth, we set the income tax rate as (iv) $\tau = 0.2$. The contribution rate for pension rises to 18.3% in Japan. Thereby, we set $\tau = 0.2$.[15] Fifth, we set the child-care cost $\hat{z}$ as (v) $\hat{z} = 0.08$, which is nearly same as the parameter that de la Croix and Doepke (2003) set.[16]

[15]The pension reform at 2004 in Japan decided that the replacement rate is kept at about a half of the working generation's wage income and the contribution rate of pension is raised step by step in every year up to 18.3% and is fixed at this contribution rate (Data: Ministry of Health, Labour and Welfare (http://www.mhlw.go.jp/topics/bukyoku/nenkin/nenkin/pdf/kaisei-h16.pdf).

[16]de la Croix and Doepke (2003) consider child-care costs not as goods cost but as opportunity costs to stop working because of child care and set $\hat{z} = 0.075$. As described by de la Croix and Doepke (2003), Haveman and Wolfe (1995) and Knowles (1999) report $\hat{z} = 0.15$. Jinno and Uemura (2008) consider not only the goods costs but also opportunity cost to have children, and set $\hat{z} = 0.2$.

Given these parametric conditions (i)–(v), the parameter set that brings about $\frac{db}{d\hat{q}} < 0$ is shown as the following table.

[Insert Table 1 around here.]

Table 1 shows that the parameter set exists that brings about $\frac{db}{d\hat{q}} < 0$ and then the steady state that public debt per capita b_t and private capital stock per capita k_t converge to certain level b and k. Theoretically, the parameter set to hold $\frac{db}{d\hat{q}} < 0$ exists. However, the fertility rate n and interest rate $1 + r$ are unrealistic values.

Therefore, we set a parameter set that is more realistic. First, the fertility rate in Japan at 2010 is 1.39. Then, we set additional parameter conditions to obtain $n = 0.7$. Furthermore, a recent interest rate is about 1% per year in Japan. One period is regarded as thirty years in the overlapping generations model and the interest rate $1 + r = (1.01)^{30} = 1.35$. Given $\hat{z} = 0.08$, $\tau = 0.2$, $\hat{x} = 0.5$, and $\hat{q} = 0$, we obtain $\alpha = 0.048$. Then, we find no steady state equilibrium under these parametric conditions, as shown in Fig. 2.[17]

[Insert Fig. 2 around here.]

[Insert Fig. 3 around here.]

Therefore, without a steady state equilibrium, the public debt stock b_t might continue increasing or decreasing, depending on the initial condition. This result is consistent with the fact that public debt in Japan is increasing.

Fig. 3-1 portrays two steady state equilibria (A and B) with parameter conditions ($\alpha = 0.2, \beta = 0.5, \hat{q} = 0.01, \theta = 0.3, \tau = 0.2, \hat{z} = 0.08$). This steady state equilibrium (point A) holds $\frac{db}{d\hat{q}} < 0$, i.e., an increase in child allowances can reduce the public debt stock per capita. However, the public debt stock is negative, i.e., the government has assets. In the steady state equilibrium, even if the primary balance is in deficit $\frac{\hat{x}}{n} + \hat{q}n - \tau > 0$, the government can continue having assets by virtue of the high interest rate. Moreover, an increase in child allowances can reduce the public debt per capita (raise the public asset per capita). An increase in the child allowance increases government expenditures directly,

[17] Given $k_0 = 0.15$ and $b_0 = 0$ as an initial condition, the private capital per capita k_1 and public debt per capita b_1 are given by $k_1 = 1.5$ and $b_1 = -1.788$ in the next period. Child allowances $\hat{q} = 0.01$ render these variables as $k_1 = 1.533$ and $b_1 = -1.7866$, which means that child allowances prevent accumulation of public assets. Given $k_0 = 0.1$ and $b_0 = 0$, the public debt per capita increases greatly. We can not derive the positive value of private capital stock per capita.

which magnifies the primary balance deficit. However, an increase in tax revenues occurs because of the increased population size.

Given the parameter condition ($\alpha = 0.3, \beta = 0.4, \hat{q} = 0.01, \theta = 0.3, \tau = 0.2, \hat{z} = 0.08$), we can depict Fig. 3-2, and an increase in child allowances can reduce the public debt stock per capita, $\frac{db}{d\hat{q}} < 0$ brings about a steady state equilibrium (point A). Point A in Fig. 3-2 shows the steady state equilibrium with $1 + r > n$. Then, the primary balance is surplus $\frac{\hat{x}}{n} + \hat{q}n - \tau < 0$. Although a high interest rate increases public debt per capita, the public debt per capita stays at a constant level because of the primary balance surplus. Moreover, the child allowance can reduce the public debt per capita even if the child allowance decreases the primary balance surplus directly. However, an increase in population size raises tax revenues; finally constant public debt per capita is maintained.

In developed countries, it might be not meaningful to examine the case of $b < 0$ because the public debt stock is positive. However, considering b as a pension fund, the case of $b < 0$ is worth examining.

However, these results are obtained in a locally unstable steady state. Therefore, it is apparent that no steady state equilibrium exists or no stable steady state that holds $\frac{db}{d\hat{q}} < 0$ exists, given by the realistic parameter condition. However, given a parameter condition that is small, different from realistic parameter, we obtain $\frac{db}{d\hat{q}} < 0$ and $b < 0$ at a locally stable steady state, as shown in Table 2.

[Insert Table 2 around here.]

(i) and (ii) in Table 2 are applied to the parameter condition $\alpha + \beta = 0.7$ and $\hat{z} = 0.08$. However, the replacement rate x and τ differ from the parameter conditions in Table 1. With small α, the stable steady state equilibrium is brought about by controlling the replacement rate x and contribution rate τ. These stable steady states are given by point B in Fig. 3-1. Furthermore, (iii) and (iv) in Table 2 have the stable steady state equilibrium given by other parameter conditions that do not hold in the condition of $\alpha + \beta = 0.7$. If β is low, implying a high household savings rate, then the stable steady state equilibrium is shown as point B in Fig. 3-2.

Yasuoka and Miyake (2012) theoretically derive that child allowances can not decrease the public debt per capita in the AK model that marginal productivity of capital is constant. In Yasuoka and Miyake (2012), child allowances always raise the public debt per capita (or reduce the public assets). Different results described in this paper and those reported by Yasuoka and Miyake (2012) are attributable to the assumption of a production function.

5 Concluding Remarks

This paper describes an endogenous fertility model with a pay-as-you-go pension model including public debt and examines how public debt is affected by the income tax rate, the pension benefit rate, and a child allowance. Given a balanced budget in each period, which does not allow public bond issuance, the government must set the income tax rate or pension benefit to balance the budget in response to an increased number of older people. However, being able to issue public debt, even if younger people become fewer or older people become more numerous, the government need not change the income tax rate and pension benefit to balance the budget as long as the pension is sustainable. Such a pension system is desired because it prevents circumstances under which a certain young generation suffers from a heavy burden or that a certain older generation suffers from low pension benefits because of a balanced budget in each period. Therefore, it is worth examination to ascertain whether such a pension system is sustainable or not.

This paper presented derivation of results for both a small open economy and a closed economy. In a small open economy, child allowances can halt the increase of the public debt stock per capita. In addition, given some parameter conditions, child allowances can reduce the public debt per capita in the steady state. However, in a closed economy, child allowances can not always stop the increase of the public debt stock per capita, as derived by numerical examples given by realistic parameter condition. Furthermore, this numerical example derives the steady state equilibrium by which child allowances can reduce the public debt per capita (or raise the public asset (pension fund)). However, given a realistic parameter, no steady state equilibrium exists or no stable steady state equilibrium exists. If the parameter condition that is not consistent with the realistic condition is given, then we have a stable steady state in which the government has negative debt, i.e., public asset and child allowances can reduce public debt (or raise public assets).

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Appendix

Appendix A: Stable Condition in a Closed Economy

The capital per capita k and public debt per capita b in the steady state are given as (13) and (14). Then, using (5), (7), (8), (9), and (12), we obtain the dynamics equation of b_t as

$$b_{t+1} = \frac{1}{n_t}\left((1+r_t)b_t + \frac{\hat{x}(b_t+k_t)}{(1-\alpha-\beta)(1-\tau) - \frac{(\alpha+\beta)\hat{x}}{1+r_t}} - \tau w_t\right) + \hat{q}w_t. \tag{21}$$

We define (12) as $F(k_{t+1}, b_{t+1}, k_t, b_t) = 0$ and (21) as $G(k_{t+1}, b_{t+1}, k_t, b_t) = 0$. Then, we derive the following equations at the approximation of the steady state, as

$$\begin{pmatrix} k_{t+1}-k \\ b_{t+1}-b \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \begin{pmatrix} k_t-k \\ b_t-b \end{pmatrix},$$

where

$$\begin{aligned} c_{11} &= -\frac{\frac{\partial G}{\partial b_{t+1}}\frac{\partial F}{\partial k_t} - \frac{\partial F}{\partial b_{t+1}}\frac{\partial G}{\partial k_t}}{C}, \quad c_{12} = -\frac{\frac{\partial G}{\partial b_{t+1}}\frac{\partial F}{\partial b_t} - \frac{\partial F}{\partial b_{t+1}}\frac{\partial G}{\partial b_t}}{C}, \\ c_{21} &= -\frac{\frac{\partial F}{\partial k_{t+1}}\frac{\partial G}{\partial k_t} - \frac{\partial F}{\partial k_t}\frac{\partial G}{\partial k_{t+1}}}{C}, \quad c_{22} = -\frac{\frac{\partial F}{\partial k_{t+1}}\frac{\partial G}{\partial b_t} - \frac{\partial F}{\partial b_t}\frac{\partial G}{\partial k_{t+1}}}{C}, \\ C &= \frac{\partial F}{\partial k_{t+1}}\frac{\partial G}{\partial b_{t+1}} - \frac{\partial F}{\partial b_{t+1}}\frac{\partial G}{\partial k_{t+1}} \end{aligned}$$

or

$$c_{11} = \frac{\frac{1+r}{n} + \frac{\hat{x}}{n^2\left(\frac{1-\tau}{n} - \frac{(\alpha+\beta)(\hat{z}-\hat{q})}{\alpha}\right)}}{1 - \frac{\alpha\hat{x}\left(\left(\tau-\frac{\hat{x}}{n}\right)w-(1+r)b\right)\frac{\partial r}{\partial k}}{n^2(\hat{z}-\hat{q})(1+r)^2\left(1-\frac{\alpha\hat{x}(1-\tau)w}{n^2(\hat{z}-\hat{q})(1+r)^2}\frac{\partial r}{\partial k}\right)}},$$

$$c_{12} = \frac{\left(b+\frac{\alpha\hat{x}^2w}{n^2(\hat{z}-\hat{q})(1+r)^2}\right)\frac{1}{n}\frac{\partial r}{\partial k} + \left(\hat{q}-\frac{\tau}{n}\right)\frac{\partial w}{\partial k} + \frac{\hat{x}\left(1-\frac{\alpha\hat{x}(1-\tau)w}{n^2(\hat{z}-\hat{q})(1+r)^2}\frac{\partial r}{\partial k}\right)}{n^2\left(\frac{1-\tau}{n}-\frac{(\alpha+\beta)(\hat{z}-\hat{q})}{\alpha}\right)}\left(1-\frac{\alpha\left(\frac{1-\tau}{n}-\frac{(\alpha+\beta)(\hat{z}-\hat{q})}{\alpha}\right)\left(\left(\tau-\frac{\hat{x}}{n}\right)w-(1+r)b\right)\frac{\partial w}{\partial k}\frac{\partial r}{\partial k}}{n^2(\hat{z}-\hat{q})(1+r)^2\left(1-\frac{\alpha(1-\tau)\hat{x}w}{n^2(\hat{z}-\hat{q})(1+r)^2}\right)^2}\right)}{1-\frac{\alpha\hat{x}\left(\left(\tau-\frac{\hat{x}}{n}\right)w-(1+r)b\right)\frac{\partial r}{\partial k}}{n^2(\hat{z}-\hat{q})(1+r)^2\left(1-\frac{\alpha\hat{x}(1-\tau)w}{n^2(\hat{z}-\hat{q})(1+r)^2}\frac{\partial r}{\partial k}\right)}},$$

$$c_{21} = -\frac{a_{11}}{1-\frac{\alpha\hat{x}(1-\tau)w}{n^2(\hat{z}-\hat{q})(1+r)^2}\frac{\partial r}{\partial k}},$$

$$c_{22} = \frac{\left(\frac{1-\tau}{n}-\frac{(\alpha+\beta)(\hat{z}-\hat{q})}{\alpha}\right)\frac{\partial w}{\partial k}}{1-\frac{\alpha\hat{x}(1-\tau)w}{n^2(\hat{z}-\hat{q})(1+r)^2}\frac{\partial r}{\partial k}} + \frac{c_{12}c_{21}}{c_{11}}.$$

We examine the following characteristic equation,

$$H(\lambda) = \lambda^2 - (c_{11}+c_{22})\lambda + (c_{11}c_{22} - c_{12}c_{21}), \tag{22}$$

Here, λ denotes the eigenvalues. The two real eigenvalues are given by the condition $(c_{11}+c_{22})^2 - 4(c_{11}c_{22}-c_{12}c_{21})$. If the following condition holds, the steady state equilibrium is obtained.

$$H(0) = c_{11}c_{22} - c_{12}c_{21} < 0, \tag{23}$$

$$H(1) = 1 - (c_{11} + c_{22}) + c_{11}c_{22} - c_{12}c_{21} > 0, \tag{24}$$

$$H(-1) = 1 + (c_{11} + c_{22}) + c_{11}c_{22} - c_{12}c_{21} > 0. \tag{25}$$

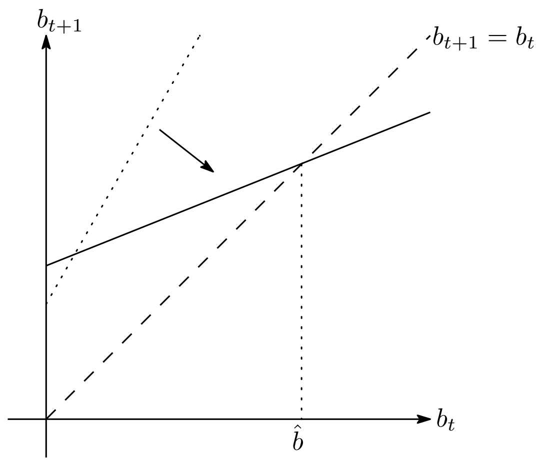

Fig. 1: Dynamics of b_t in a Small Open Economy.

α	β	q	db/dq	k	1+r	n
0.2	0.5	0.01	−0.00385	0.01541	5.56749	2.542306
0.2	0.5	0.02	−0.07259	0.01377	6.023758	2.943349
0.2	0.5	0.03	−0.43169	0.01353	6.098356	3.527957
0.3	0.4	0.01	−0.0756	0.0081	8.733368	3.673936
0.3	0.4	0.02	−0.1304	0.00774	9.015765	4.277292
0.4	0.3	0.01	−0.0792	0.00534	11.69075	4.815822
0.4	0.3	0.02	−0.15388	0.00551	11.43708	5.624783

Table 1: Parameter Set to Hold $\frac{db}{d\hat{q}} < 0$

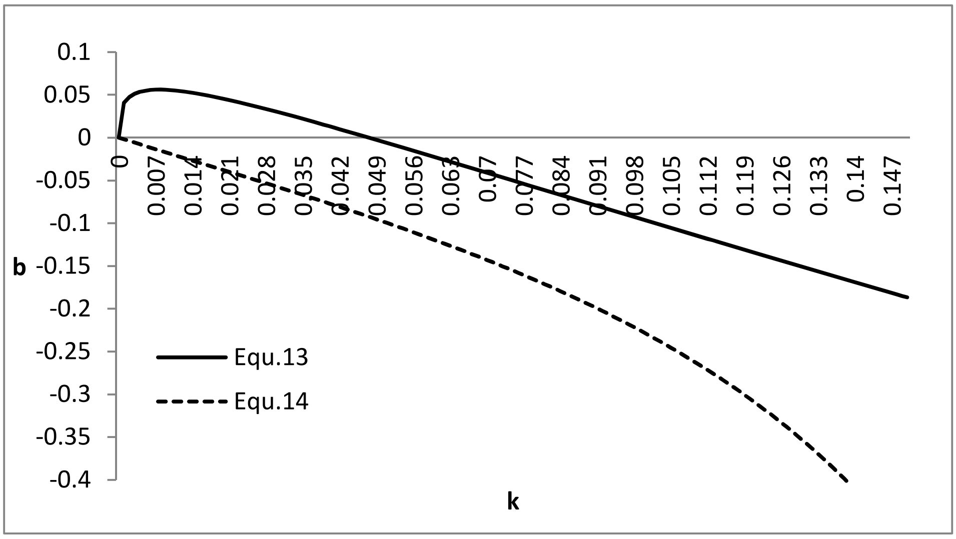

Fig. 2: No Steady State Equilibrium.

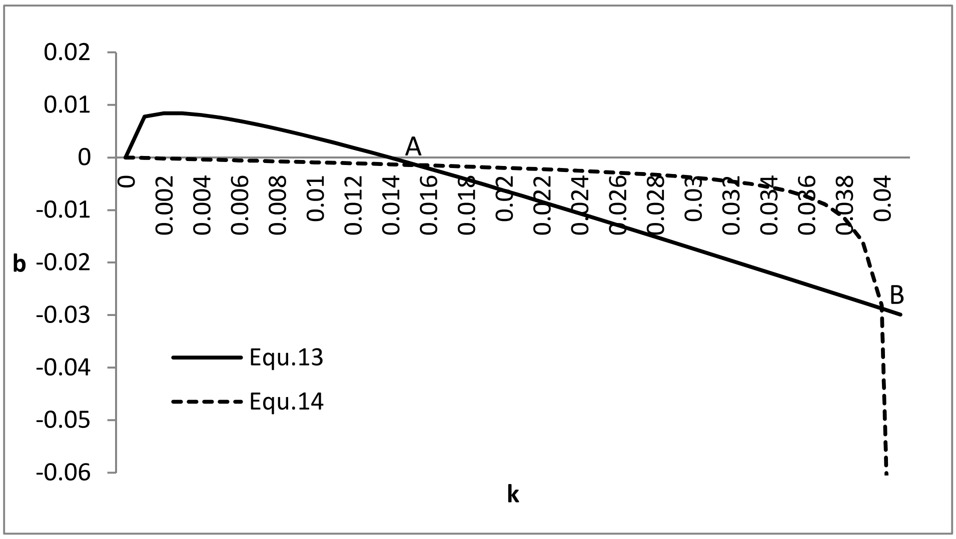

Fig. 3-1: Two Steady State Equilibrium (Negative Public Debt).

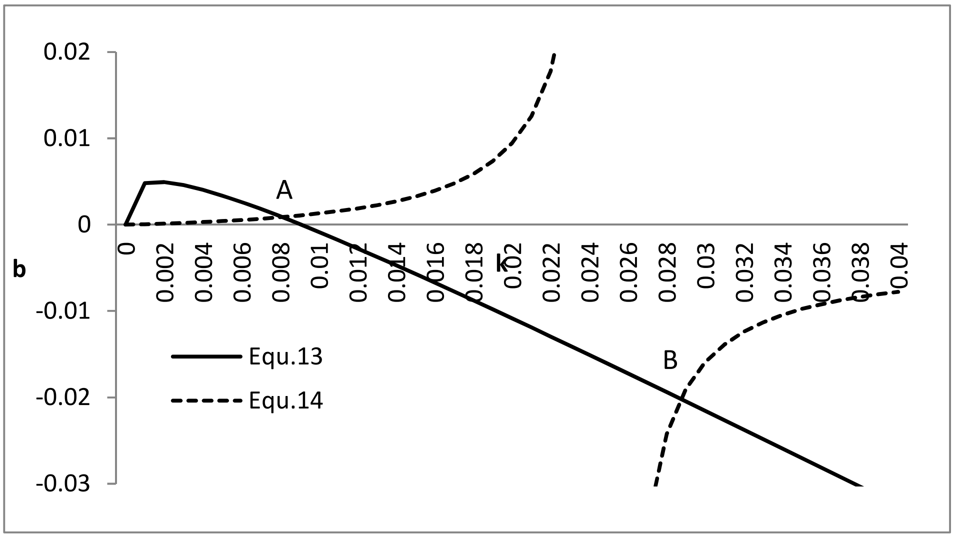

Fig. 3-2: Two Steady State Equilibrium (Positive Public Debt).

	α	β	z	q	x	τ	db/dq	k	1+r	n
i	0.1	0.6	0.08	0.02	0.4	0.2	−1.2337	0.051228	2.401409	1.610948
ii	0.1	0.6	0.08	0.03	0.7	0.3	−0.2317	0.039059	2.903464	1.882183
iii	0.1	0.1	0.08	0.04	0.7	0.4	−1.4146	0.062478	2.089836	2.337386
iv	0.1	0.1	0.08	0.04	0.9	0.5	−0.7118	0.064259	2.04912	2.348032

Table 2: Parameter Set to Hold $\frac{db}{d\hat{q}} < 0$ in a Stable Steady State.